JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 31, No. 6, November–December 1994

Application of Probabilistic Analysis and Design Methods in Space Programs

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The use of probabilistic analysis and design methods in space programs is steadily gaining acceptance within the aerospace community. To further enhance the development and application of probabilistic methods, four actions are recommended: 1) the development of theories, codes, and tools to match the task areas; 2) an understanding of prior applications/uses of probability and statistics, and databases; 3) the development of databases, in particular structural failures; and 4) education of engineers and managers on the merits and basics of probability/reliability methods. The approaches, the status, and the needs for these actions are examined in terms of the current limitations of probabilistic engineering methods, a basic approach for its application, a discussion of some specific uses, and recommendations of critical development areas.

Nomenclature

dA/dN = crack growth per cycle
G_{rms} = root-mean-square acceleration, g

Introduction

PROBABILITY and statistics have been used extensively in prior and ongoing space programs in specific areas, but have not had a universal focus or acceptance. Where used, probabilistic methods have contributed significantly to the success of the programs. In recent years, the research focus has been to extend these methods for application in design. In fact, NASA design requirements for future space vehicles and structures will be specified in reliability terms. However, making the transition from using probabilistics and statistics in engineering analysis to the design of structures for reliability is very difficult, due to the limited structural databases available in aerospace, and the complexity of the dominant failure modes of wear, fatigue, fracture, and buckling. Nevertheless, probabilistic analysis/design is becoming a viable approach in all aspects of space programs. Many approaches are currently in use, and development is underway on more encompassing techniques such as numerical evaluation of stochastic structures under stress (NESSUS) and probabilistic risk assessment (PRA). Developing strategies for advancing the application of probabilistic methods within aerospace provided the impetus for this paper.

Uncertainties in the definition of loads and environments, material properties, geometric variables, manufacturing processes, engineering models, analysis tools, etc., and all types of testing, including development, verification, and certification, lead to uncertainties in space vehicle and structural design, and ultimately safety. Clearly, quantifying and understanding problem uncertainties and their influence on design variables develops a better-engineered, better-designed, and safer system. Two formats are available for characterizing design uncertainties: 1) deterministic/safety factor, and 2) probabilistic/reliability. The classical deterministic analysis approach accounts for design uncertainties in "lump sum" fashion by multiplying the maximum expected applied stress (load, environment, etc.), a single value, by a factor of safety. Often, design verification is achieved by applying a worst-case loading (e.g., a 3σ load condition multiplied by the safety factor) to the structure and testing to failure. In areas other than structures the same approach can be used. In contrast, the probabilistic format attempts to model individual parameter uncertainty by a probability distribution. A test-constructed database gives the best characterization. If test data are not available, then the engineer must make assumptions concerning the parameter distributions. Once the distributions are defined, model equations are used to combine the density functions into a cumulative distribution function of the design variable—for example, applied stress. In this case, the design parameter has an uncertainty that is quantified in terms of risk.

In the case of the design of aerospace structures, it is impossible in the near future to develop an aerospace structural design code based solely on the probabilistic format without compromising the structural safety of hardware design. This statement is especially true with respect to the currently available probabilistic engineering analysis tools and test verification programs. In general, the analytical tools that have been developed are difficult to understand and to implement in a design procedure; and more importantly, the methods have not been test-verified or universally accepted by the engineering community. Before a probabilistic-based design code or program can be successful, design engineers must develop an experience and education base in this field and also accumulate adequate failure databases. Today, most engineering schools do not offer probabilistic-based design courses as part of the curriculum. This does not mean, however, that these approaches do not have merit. They serve as excellent tools for assessing design concepts using sensitivity analysis and trade studies. Other design disciplines such as avionics have substantial databases, and therefore are using probabilistics in many ways in the design and verification of their hardware.

True reliability must be demonstrated, not simply estimated from an engineering analysis. Until failure and failure-rate databases are available from experience, probabilistic methods can best be utilized as a design tool to help identify the sensitivities of problem parameters. Furthermore, "demonstrated structural reliability" is virtually an impossible task due to the expense and small number of aerospace structures that are built. (This is not true in avionics.) However, it may be feasible to develop a more consistent structural design code that uses the probabilistic format in combination with the accepted safety-factor approach to design. The civil engineering profession has successfully used a combined format in the development of the load and resistance factor design (LRFD) code for steel structures.[1] Developing a similar concept for application within aerospace offers a practical area for future research. Before exploring these ideas further it is prudent to look at some areas in addition to avionics that have successfully used probabilistics. Three examples are:

Presented as Paper 93-1381 at the AIAA/ASME/ASCE/AHS/ASC 34th Structures, Structural Dynamics, and Materials Conference, La Jolla, CA, April 19–22, 1993; received May 12, 1993; revision received Dec. 29, 1993; accepted for publication Jan. 3, 1994.

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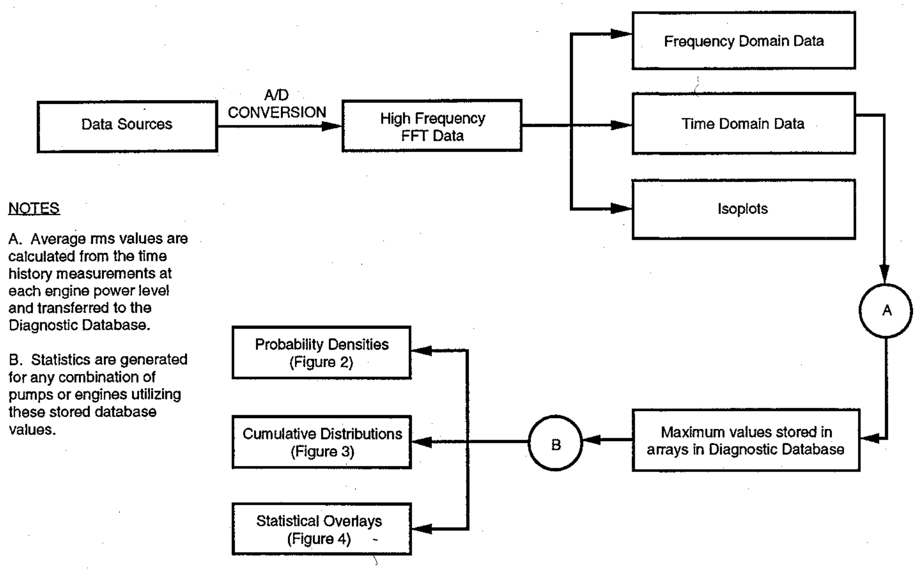

Fig. 1 Statistical processing of SSME high-frequency data.

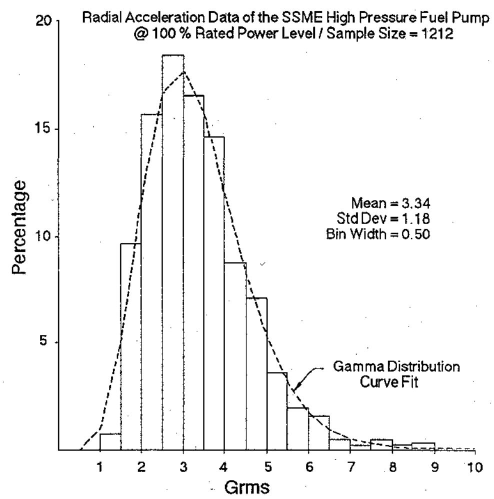

Fig. 2 Probability density plot of SSME engine data.

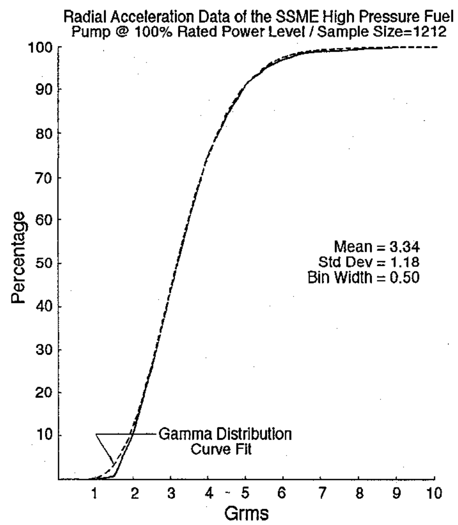

Fig. 3 Cumulative-distribution probability plot of SSME engine data.

1) component development, qualification, and acceptance in shock and vibration, 2) liquid-propulsion-engine vibration and fatigue, and 3) control, dynamic response, and loads.

Previous Applications

Components

All aerospace system components are designed, qualified, and accepted using probabilistic approaches. Their design in general is driven by shock and vibration environments that have their source in mechanical or acoustical excitations. Shock is also a source of these excitations. Because these high-frequency environments are nearly impossible to formulate analytically, extensive databases have been developed for both the excitation and the response of basic structural types with different types (and sizes) of components. Input and output responses are put in probabilistic format to serve as a base for formulating design, qualification, and acceptance criteria. Using these criteria, shock and vibration tests are run on each component. In special cases, all-up acoustical tests are run as further verification of the system, using the probabilistic acoustical environment as input. The shock-and-vibration discipline has become very successful using this approach; that, however, has been accomplished through a universal effort to establish databases, special instrumentation, data evaluation, and basing techniques.

Dynamic Engine Data

The Space Shuttle main engine (SSME) has had an extensive program to collect dynamic (also performance) data statistize them, and use them for structural durability, turbomachinery health, and maintenance and refurbishment of hardware. This database is cataloged by engine number, part number, test number (or flight), and test stand. Most firings and flight are over 500 s in duration, and the frequency content of interest ranges up to 3000 Hz. Figure 1 illustrates the basic approach for the statistical processing of the data. Obviously, this creates a very large database requirement as well as the need for fast processing schemes and user-friendly access to the data. Figures 2, 3, and 4 are typical plots of data outputs. By combining all test and flight data including test failures, it is possible to say statistically what constitutes healthful hardware and what is used to determine good hardware during green runs (certification), as well as when to change out parts. In addition, the engine has been mapped into vibration zones, and acceleration data acquired for use to determine loads for hardware assessment and redesigns. These data can also be used as a starting point for future engine-system design.

Dynamic Responses/Loads

Probabilistic approaches have been used extensively in the determination of launch-vehicle control and dynamic responses and loads. The environments that produce these responses include both natural and induced environments. The aerospace community has developed a natural-environments database that is very extensive, including atmospheric density, temperature, winds, solar pressure, etc. This statistical database serves as one distribution into the response

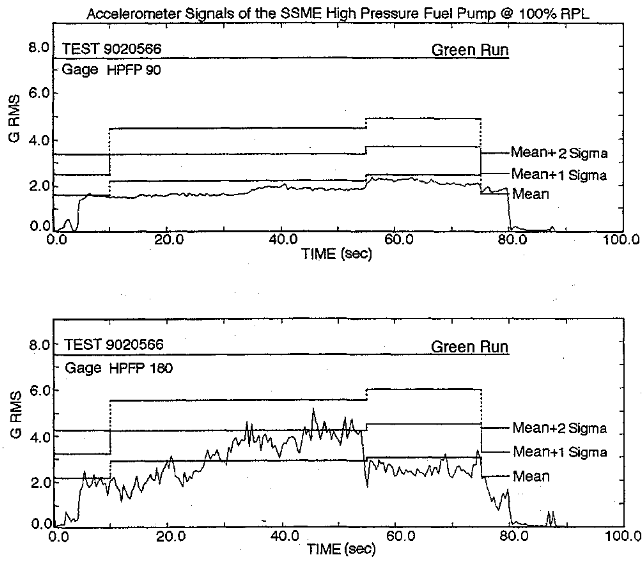

Fig. 4 Statistical information on SSME engine data.

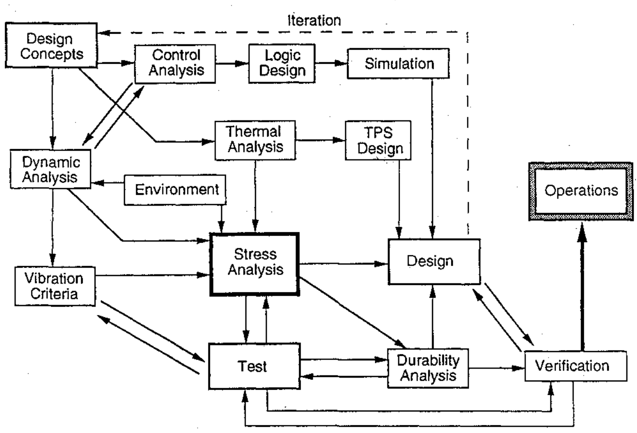

Fig. 5 Generic flow for structural analysis.

analysis. With regard to propulsion-system characteristics, the same rigor has been applied to the collection of data on thrust, thrust rise rate, oscillations, and thrust-vector misalignments to serve as inputs for these approaches. Aerodynamics, another induced environment is investigated with wind-tunnel testing and computational fluid dynamics (CFD). The resulting analysis (loads) can be accomplished using deterministic approaches or probabilistics, depending on the needs. Figure 5 is a flow diagram showing how this is accomplished for structural analysis. This database, in conjunction with day-of-launch wind measurements, etc., can be used to insure a safe launch.

Basic Approach

The basic probabilistic approach can be summarized as the quantification of all input data, the model equations, and the output in a statistical manner. This requires the use of a statistical procedure to take the model equations and the input data and produce a statistical/probabilistic output. These techniques range from pure Monte Carlo to integral solutions. Figure 6 illustrates this process for structural assessment. The left-hand side of the figure shows all of the input data, indicating a statistical distribution for each. The right-hand side illustrates the various capabilities of the structure, and the middle shows the output of the process again in a statistical sense.

Nothing has been said of how the input or capability data are generated, nor of how the model equations are solved to get the stress or capability distribution. Many techniques exist to accomplish this task. The describing equations can be solved using Monte Carlo approaches by inputting the parameters as statistical distributions and then solving the equations for the various combinations selected randomly. The output becomes a probability distribution function or statement. The same equations can be solved using the A-factor approach[2] and the 3σ limits of each parameter. The A-factor approach allows the generation of 3σ equivalent time response analysis by root-sum-squaring the Δ's for each parameter variation. A Δ value is defined as the change in the output parameter from its nominal value due to a 3σ change in a single input parameter. The root-sum-squared value is used with the individual Δ's as a ratio to apply to the model equation coefficients. The final output is a 3σ answer.

Other techniques exist to deal with such data.[3–37] For all, the objective is to rate the probability of an event occurring against one's capability of dealing with that occurrence. This means understanding, and predicting the failure modes of capabilities. Given that these can be accomplished, then it is straightforward to know how failures occur, etc. Whether one uses Bayesian statistics or any of many other

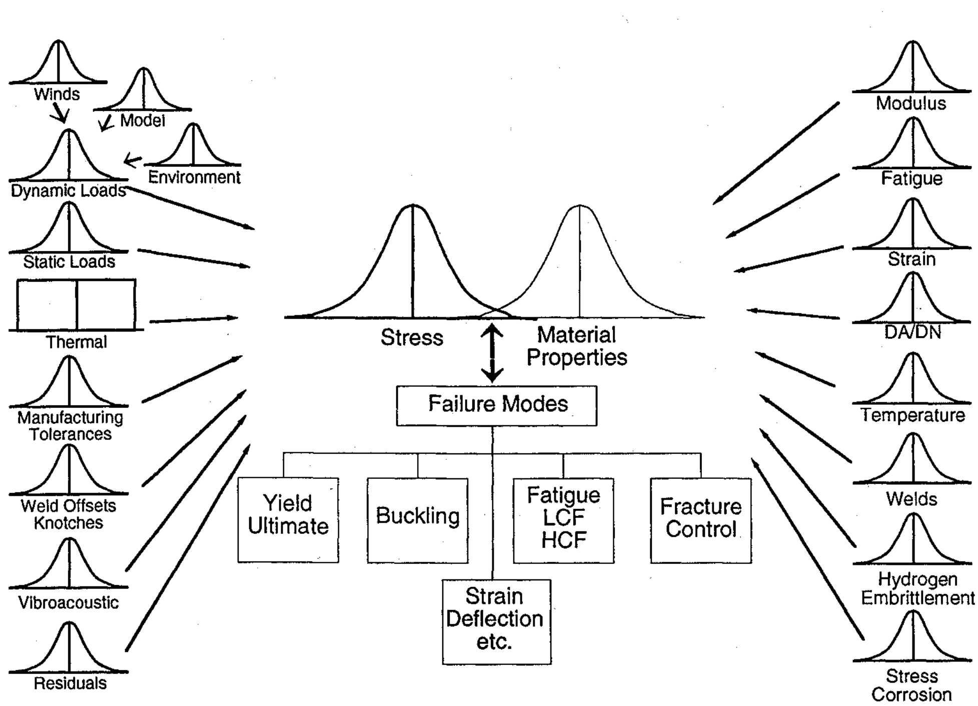

Fig. 6 Probabilistic analysis concept.

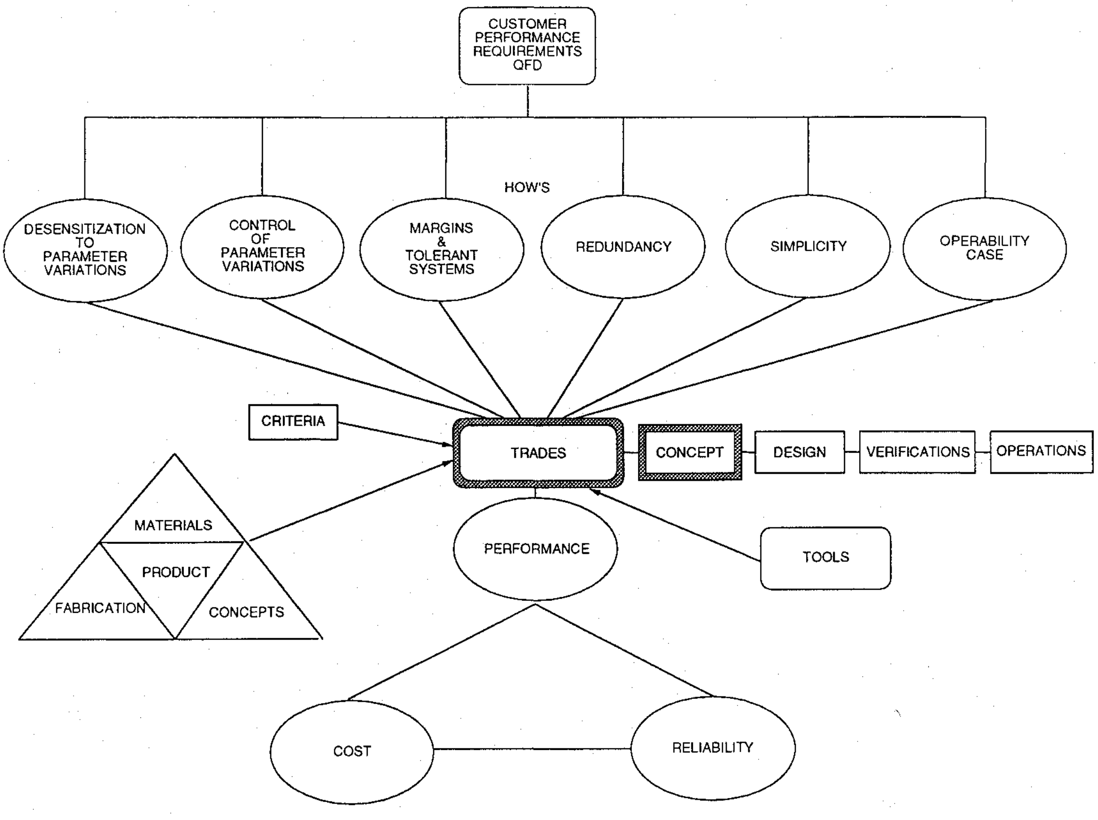

Fig. 7 Robustness design approach.

tools, key statements can be made concerning the reliability of the system when the data or good estimations are available. This same sequence of flow takes place for the deterministic assessment; in that case, however, no real probabilistic or reliability statement can be made concerning the outcome. Ideally, a probabilistic statement is highly desirable. In practice, this may not be possible; however, it is prudent to utilize as much statistical information as possible about the characteristics of the system. This means that in reality a blend between the deterministic and the probabilistic must be used.

In the past, emphasis has been placed on the tools for solving the model equations to provide a statistical answer and ways to estimate the input data in some distribution sense. All of this is needed; however, if the complete statistical approach is to become an accepted practice, data must be acquired in terms of failures, inputs, etc., as has been accomplished in the three examples given previously. For the present, use must be made of all the available data in a combined manner as discussed.

This combined approach uses the statistical data available for the various parameters and combines them into a mean and standard deviation, which is used in conjunction with the concept of a deterministic safety factor.[37] Another version of the approach uses best estimates of the parameter variations and then conducts sensitivity analysis and trades to arrive at concept selections and design and operational approaches. The assumed parameter distributions can be varied and become part of the sensitivity analysis. Figure 7 illustrates this approach as applied to the design of a system for robustness. Notice the areas of robustness, the trade criteria, and the product parts that must be party to this trade. Probabilistic/reliability data are the key to making the right choices.

Another area of quality/safety is failure modes and effects analysis, which leans heavily on the probabilistic approach. The end assessment is always "How much risk are you taking?" Fundamental to this analysis is the fault tree, events tree, or logic tree.

Current Limitations of Probabilistic Engineering Methods

The ultimate goal of probabilistic engineering application is to define accurately the reliability of a given design. Although this goal is desirable, it is a very difficult one to achieve for complex structural problems, not only in terms of engineering cost and verification, but also in terms of problem understanding. The major limitations of current probabilistic engineering tools center about this basic goal. Some of the more general ones are summarized below.

1) Many of the probabilistic analysis tools that have been developed are problem-specific and, to work on a given failure problem, require software modifications. In some cases this type of approach seems unavoidable, especially for extremely complex problems that are highly nonlinear and require several mathematical models and iterations simply to get one deterministic value. However, practicing engineers are interested primarily in tool application for use in understanding design problems. Tool development is not their specialty.

2) Generic tools, like NESSUS, do not interface directly with general structural-analysis software packages such as NASTRAN, ANSYS, etc. Before probabilistic methods will be universally accepted or even experimented with in any great detail, they must interact freely with at least one well-developed analysis/design tool presently utilized by the practicing engineer.

3) The probabilistic format requires distribution information for analysis input. In most cases, adequate databases are not available, and the engineer must make assumptions regarding the data. Guidelines for this phase of the process need to be developed. Always, the analysis should be checked with different inputs to verify the effects of distribution assumptions.

4) Like structural finite-element models, probabilistic mathematical models are only approximations to the real world. Model inaccuracies and their effects on design must be determined. The accuracy of any engineering analysis is only as good as the assumptions that define it.

5) Computational schemes are grouped into two categories: a) simulation, and b) approximate reliability methods, such as first-order reliability method (FORM), second-order reliability method (SORM), advanced mean value (AMV), etc., that use fast probability integration (FPI) techniques. While Monte Carlo simulation has fewer assumptions, it requires extensive and expensive computer time for large complex models. "Efficient" Monte Carlo simulation

methods have been developed, and these, along with faster computers, make simulation a valuable application tool for many simpler problems. Although the approximate methods are faster and more efficient with respect to computer usage, they involve defining explicit response functions that can be difficult to verify in terms of accuracy and convergence. Inaccuracies can arise from poor approximations to the true response function and nonlinear coordinate transformations.

Many of the limitations noted are related to accuracy. While accuracy is important, it should not be the controlling factor limiting the application of probabilistic methods. The methods themselves lead to a better approach to design, because in that they require that the engineer understand more details about the problem, and even other disciplines. Not only does the probabilistic engineering approach give the designer an idea of the risk defined by a given design, it also provides a glance at the parameter design sensitivities. In general, probabilistic methods require more detailed analysis, which ultimately can lead to a more robust design.

Recommendations of Critical Development Areas

It is very difficult to make the jump from using probability and statistics in engineering analysis to predicting the reliability of structures. First of all, there is a limited structural database, and to develop failure and failure-rate databases for general use in space programs is far too expensive. Second, the dominant failure modes of wear, fracture, fatigue, and stability are very complex, leading to model and analysis inaccuracies. Therefore, for the near term, the goal should not be to replace current engineering design practices with probabilistic methods. Rather, the goal should be to supplement current safety-factor deterministic approaches with probabilistic methods. For example, performing future designs using both probabilistic and deterministic approaches in parallel will help to calibrate the probabilistic methods. The emphasis should be on the gradual introduction of probabilistic methods that focus on sensitivity analysis at the component/failure-mode level: 1) to characterize and determine the effects of loads and material property uncertainties and 2) to estimate the risk of particular failure modes, such as fatigue and crack propagation. This approach is of relevancy to the practicing design engineer and ultimately can lead to a more robust design. The greatest strength of the probabilistic design approach is its use in determining the sensitivity of life drivers, and the greatest weakness is its inability to demonstrate or verify the reliability predictions of expensive aerospace structures.

At the present stage of probabilistic analysis development in aerospace applications, it is extremely important to get into the engineer's hands a probabilistic analysis tool that easily interfaces with his current structural analysis tools (i.e., finite elements). NESSUS,[38] a probabilistic finite-element software package developed by Southwest Research Institute through a NASA Lewis research grant, is this kind of tool. However, it needs to be more user-friendly and flexible so that engineers can experiment with probabilistic analysis without too much difficulty. If a user-friendly and flexible application tool is not provided to the engineer, then probabilistic methods will be used only in research programs and not in design applications.

Additional recommendations that will help enhance the development and application of probabilistic methods are:

1) Structural design and analysis laboratories need to be configured away from their traditional matrix-type organization. Materials and loads engineers (static and dynamic), designers, stress analysists, thermal and acoustics engineers, etc., should work together on a specific problem under one project supervisor. Probabilistic analysis requires detailed characterization of information (which requires time and money) and communication between engineers. Organizational boundaries can only impede the advancement of probabilistic analysis/design methods, and hence, they must be made very flexible or else removed.

2) Engineers must be educated and trained in probabilistic analysis theory and application. The aerospace community must request that universities include reliability engineering courses as requirements in undergraduate curriculums. Engineers entering the work force need to be familiar with the fundamental principles.

3) A working group should be established across the aerospace community to advance probabilistic methods. Developing a plan for integrating probabilistic analysis/design methods into space-program design practices should be the charter. Consideration should be given to application, tool, and database development, and to design codes that utilize both probabilistic and deterministic methods.

4) Studies need to be completed on existing aerospace structures to help identify and gauge the current balance of risk against safety. For hardware that has experienced failure in field applications, a probabilistic analysis should be completed before redesign is implemented.

5) Research programs should focus on developing methods for integrating limited test information into the probabilistic design/analysis process—in particular, verification of probabilistic analysis and reliability estimates with limited test data.

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